Hi-Tech Institute of Engineering & Technology	
DEPARTMENT OF Applied Sciences	
MODEL QUESTION PAPER, ODD SEMESTER-2023-24,	
Semester:1st	**Course/Branch: B.Tech.**
Subject Code:BAS103	**Subject Name: Mathematics -1**
Faculty Name: Dr. Ashfaq Ahmad, Dr. Vijay Sharma	
Time: 3:00 Hours	***Total Marks: 100***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION-A

1. Attempt all question in brief. **2x 10 =20**

Q. No	Question	Marks	CO
a.	Define Leibnitz theorem	2	2
b.	State Green 's Theorem.	2	5
c.	State Duplication formula.	2	4
d.	Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$	2	1
e.	Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$	2	1
f.	Evaluate $\int_0^1 x^2 \ (1-x)^3 \, dx$.	2	4
g.	Evaluate $\int_0^1 dx \int_0^{x^2} x \, dy$.	2	4
h.	Show that $\vec{F} = \left(x^2 - y^2 + x\right)\hat{i} - \left(2xy + y\right)\hat{j}$ is irrotational.	2	5
i.	State Taylor's Theorem.	2	3
j.	Find the Value of Γ(-1/2) .	2	4

SECTION-B

2. Attempt any three parts of the following: **3x10 =30**

Q. No	Question	Marks	CO
a.	Verify Cayley Hamilton Theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and hence find A^{-1}	10	1
b.	If $x^x y^y z^z = c$, show that at $x = y = z$, $\dfrac{\partial^2 z}{\partial x \partial y} = -\left(x \log ex\right)^{-1}$	10	2
c.	Verify whether the following functions are functionally dependent, and if so find the relation between them $u = \dfrac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$	10	3
d.	Show that $\Gamma n \Gamma(1-n) = \dfrac{\pi}{\sin n\pi}$, $0 < n < 1$	10	4
e.	If $\vec{F} = x^3\hat{i} + y\hat{j} + z\hat{k}$ is the force field. Find the work done by $\vec{F}$ along the line from (1,2,3) to (3,5,7) .	10	5

SECTION-C

3. Attempt any ONE part of the following: **1x10 = 10**

Q. No	Question	Marks	CO

a.	**Find the inverse by elementary row transformation** $A=\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}$	10	1
b.	**Determine for what value** λ **and** μ **the following equation** $x+y+z=6$ $x+2y+3z=10$ $x+2y+\lambda z=\mu$ **Have (i) No solution (ii) a unique solution (iii) infinite number of solutions.**	10	1

4. Attempt any ONE part of the following: **1x10 = 10**

Q. No	Question	Marks	CO
a.	**If** $y=(\sin^{-1}x)^2$ **prove that** $(y_n)_0=\begin{cases}0 & n\ \text{is odd}\\ 2.2^2.4^2.6^2........(n-2)^2 & n\ \text{is even}\end{cases}$	10	2
b.	**If** $z=x^2\tan^{-1}\frac{y}{x}+y^2\tan^{-1}\frac{x}{y}$ **prove that** $\frac{\partial^2 z}{\partial x\partial y}=\frac{x^2-y^2}{x^2+y^2}$	10	2

5. Attempt any ONE part of the following: **1x10 = 10**

Q.No	Question	Marks	CO
a.	**If u,v,w are the roots of** $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$**, cubic in** λ**, find** $\frac{\partial(u,v,w)}{\partial(x,y,z)}$	10	3
b.	**A ballon is in the form of right circular cylinder of radius 1.5 m and length 4m and is surmounted by hemispherical ends, if the radius is increased by 0.01 m and the length is 0.05 m, find the percentage change in the volume of the ballon.**	10	3

6. Attempt any ONE part of the following: **1x10 = 10**

Q. No	Question	Marks	CO
a.	**Prove that** $\int_0^{\pi/2}\sin^p\theta\cos^q\theta d\theta=\frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$	10	4
b.	**Find the mass of an octant of the ellipsoid** $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ **the density at any point being** $\rho=kxyz$	10	4

7. Attempt any ONE part of the following: **1x10 = 10**

Q.No	Question	Marks	CO
a.	**Prove that** $\vec{F}=(y^2-z^2+3yz-2x)\hat{i}+(3xz+2xy)\hat{j}+(3xy-2xz+2z)\hat{k}$ **is both solenoidal and irrotational.**	10	5
b.	**Use the Divergence theorem to evaluate** $\iint_S xdydz+ydzdx+zdxdy$**, where S is the portion of the plane** $x+2y+3z=6$ **which lies in the first octant.**	10	5