Hi-Tech Institute of Engineering & Technology	
DEPARTMENT OF APPLIED SCIENCES	
MODEL TEST PAPER, ODD SEMESTER-2023-24	
Semester: B.Tech Ist year, I semester	Course/Branch: Sec A/B/C/D/E/F (All Branches)
Subject Code: BAS 103	Subject Name: Engineering Mathematics I
Faculty Name: Dr. Neenu Gupta, Mr. Vijay Kumar Sharma, Dr. Ashfaq Ahmed	
Time: 3:00 Hours	*Total Marks: 70*

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION-A

1. Attempt all question in brief. **2x 7 = 14**

Q.No	Question	Marks	CO
a.	Find the eigen value of A^3-2A+I where $A=\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$	2	1
b.	Prove that the matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1+i\\ 1-i & -1\end{bmatrix}$ is unitary matrix	2	1
c.	If $y=\sin\left(m\sin^{-1}x\right)$then show that $\left(1-x^2\right)y_2-xy_1+m^2y=0$	2	2
d.	If $V=\left(2x-3y,3y-4z,4z-2x\right)$, compute the value of $6V_x+4V_y+3V_z$	2	2
e.	An error of 2% is made in measuring length and breadth then find the percentage error in the area of the rectangle.	2	3
f.	Change the order of integration $\int_0^1\int_{x^2}^{2-x} f(x,y)dydx$	2	4
g.	Find the velocity potential ϕ, such that $\nabla\phi=\left(y\sin z-\sin x\right)\hat{i}+\left(x\sin z+2yz\right)\hat{j}+\left(xy\cos z+y^2\right)\hat{k}$	2	5

SECTION-B

2. Attempt any THREE part of the following: **3x7 =21**

Q.No	Question	Marks	CO
a.	Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$ and hence find its inverse.	7	1
b.	If $y=e^{a\cos^{-1}x}$, then show that $\left(1-x^2\right)y_{n+2}-\left(2n+1\right)xy_{n+1}-\left(n^2+a^2\right)y_n=0$. Also find $y_n(0)$	7	2
c.	Obtain Taylor's expansion of $\tan^{-1}\frac{y}{x}$ about $(1,1)$upon and including the second degree terms. Hence compute $f(1.1,0.9)$	7	3
d.	Using the transformation $x+y=u$ and $y=uv$, show that $\iint\left[xy(1-x-y)\right]^{1/2}dxdy=\frac{2\pi}{105}$, integration being taken over the area of triangle bounded by the lines $x=0$, $y=0$, $x+y=1$	7	4

e.	**Verify Stoke's theorem for** $\vec{F}=(y-z+2)\hat{i}+(yz+4)\hat{j}-xz\hat{k}$ **over the surface of the cube** $x=0$, $y=0$, $z=0$, $x=2$, $y=2$, $z=2$ **in above XOY plane.**	**7**	**5**

SECTION-C

3. Attempt any ONE part of the following: **1x7 = 7**

Q.No	Question	Marks	CO
a.	**For what value of k , the equations** $x+y+z=1$ $2x+y+4z=k$ $4x+y+10z=k^2$ **Have a solution and solve them completely in each case.**	**7**	**1**
b.	**Find the eigen values and corresponding eigen vectors of the matrix A** $A=\begin{bmatrix}-17 & 18 & -6\\ -18 & 19 & -6\\ -9 & 9 & 2\end{bmatrix}$	**7**	**1**

4. Attempt any ONE part of the following: **1x7 = 7**

Q.No	Question	Marks	CO
a.	**If** $u=\sin^{-1}\left(\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}\right)^{1/2}$ **, Show that** $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial z^2}=\frac{\tan u}{144}\left(13+\tan^2 u\right)$	**7**	**2**
b.	**Trace the curve** $y^2(a+x)=x^2(3a-x)$	**7**	**2**

5. Attempt any ONE part of the following: **1x7 = 7**

Q.No	Question	Marks	CO
a.	**If** $y_1=\frac{x_2x_3}{x_1}$, $y_2=\frac{x_1x_3}{x_2}$, $y_3=\frac{x_2x_1}{x_3}$ **then evaluate** $\frac{\partial(x_1,x_2,x_3)}{\partial(y_1,y_2,y_3)}$	**7**	**3**
b.	**Find the dimension of rectangular box, without top of maximum capacity whose surface area is 108 sq. cm.**	**7**	**3**

6. Attempt any ONE part of the following: **1x7 = 7**

Q.No	Question	Marks	CO
a.	**Find the volume of the region bounded by the surface** $y=x^2$, $x=y^2$ **and the plane** $z=0$, $z=4$	**7**	**4**
b.	**Show that** $\iiint\frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}=\frac{\pi^2}{8}$ **the integral being extended to all positive values for which the expression is real.**	**7**	**4**

7. Attempt any ONE part of the following: **1x7 = 7**

Q.No	Question	Marks	CO
a.	**Find the directional derivative of** $f(x,y,z)=2x^2+3y^2+z^2$ **at the point** $P(2,1,3)$ **in the direction of the vector** $\vec{\alpha}=\hat{i}-2\hat{k}$	7	5
b.	**Verify Gauss Divergence theorem for** $\vec{F}=(2x-z)\hat{i}+x^2y\hat{j}-xz^2\hat{k}$ **taken over the rectangular parallelopiped** $0\le x\le a$, $0\le y\le b$, $0\le z\le c$	7	5